Transconductance as a Criterion of Electron Tube Performance

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QUANTITATIVE evaluation of electron tube performance has assumed added importance with the increasing extension of electronics into the fields of measurement and control. Simplification of the process of selection of suitable tube types and operating conditions from the general data available is of considerable value to all engineers concerned with electronics circuit design. The conventional procedure involves analysis of the plate current-grid voltage characteristics. The simpler method presented herein supplies the same information from an analysis of the transconductance-grid voltage characteristics. These are usually supplied by the manufacturer or can be obtained readily by measurement[1].

The method presented herein has been employed successfully for a number of years in the development of electronic measuring apparatus by a group of engineers who attended lectures on the subject given by the author. It applies chiefly to pentodes, where the internal plate impedance is high with respect to the load impedance. Its merit resides in the comparative brevity of the formulae, the ease of computation and the facility in obtaining the data from which the computations are made. It allows one to form a preliminary judgment of the performance of a tube from a brief glance at the characteristics furnished by the manufacturer better and faster than any other method known to the author. It should prove of value to the instructor teaching electron tube theory.

In the interest of simplicity some of the subscripts *m*, *p*, *c* and *g* appended usually to symbols for transconductance, plate current and grid voltages are deleted below. The scope of the discussion is so limited that no confusion may arise from this omission. The formulae are expressed in terms of amplitudes of voltage and current, capital letters being used for their symbols. All values are in peak volts. Levels are in decibels.

The *g*-*e* characteristic is introduced into the problem by starting with the general expression for the plate current

$$i = i_0 + \frac{\partial i}{\partial e} v + \frac{1}{2!} \frac{\partial^2 i}{\partial e^2} v^2 + \frac{1}{3!} \frac{\partial^3 i}{\partial e^3} v^3 + \cdots \tag{1}$$

where v is the voltage measured from the bias point E_c, where the derivatives are taken, and utilizing the definition of the transconductance

[1] Radio Engineers' Handbook, F. E. Terman, McGraw-Hill, 1943, p. 961.

$$g = \frac{\partial i}{\partial e}. \tag{2}$$

Inserting (2) into (1) and calling G the value of g at E_c we obtain

$$i = \int_{-\infty}^{E_c} gde + Gv + \frac{1}{2}\frac{\partial g}{\partial e} v^2 + \frac{1}{6}\frac{\partial^2 g}{\partial e^2} v^3 + \frac{1}{24}\frac{\partial^3 g}{\partial e^3} v^4 \cdots \tag{3}$$

The first term of this expression is the space current of the tube at no load, that is when $v = 0$. On the g-e diagram, Fig. 1, it represents the area under the curve from the tube cut off C to the tube bias E_c. The second term represents the function of the tube as an amplifier. The third term represents the second-order modulation current. The latter is responsible for the objectionable generation of a second harmonic in an amplifier and the useful presence of the second harmonic in the frequency doubler, the direct current in a rectifier and the sidebands in a modulator.

The fourth and higher terms represent, in general, undesirable effects of modulation. They are usually smaller than the first two and, since their effects are additive, the first three terms of expression (3) may be studied profitably disregarding the others. If necessary, the effects of the higher-order terms may be added later.

The Idealized Parabolic Pentode

If, over a certain range of grid biases e_A to e_B, the effect of the fourth and higher terms of series (3) is negligible the g-e characteristic will be a straight line. Herein lies one of the advantages of the method, for a straight portion of a curve can be easily selected by inspection and checked with a straight edge. It is thus possible to select easily such a tube and operating point that third and higher-order modulation products are absent in the output. There is no such simple method of verifying whether a current characteristic is parabolic. That there are tubes having approximately straight portions of g-e characteristics can be verified by inspection of (Fig. 1) where the characteristic of the 6AG7 is given.

Since a portion of the g-e characteristic is a straight line, the third term coefficient $\frac{\partial g}{\partial e}$ may be replaced by the ratio $\frac{\Delta g}{\Delta e}$ where Δe is an arbitrary interval of grid voltage and Δg the corresponding change in g. In many of the following computations it will be advantageous to use for Δe the total excursion of the grid voltage.

On the basis of the simplifying assumption of a parabolic pentode it is possible to derive the simple formulae given below which cover the performance of the tube as an amplifier, rectifier and modulator.

THE PARABOLIC PENTODE AMPLIFIER

Over the straight portion of the *g-e* curve the following relations hold for an input $v = P \cos pt$.

The fundamental current is $I_p = GP$ where P is the grid swing.

The second harmonic current is

$$I_{2p} = \frac{1}{4} \frac{\Delta g}{\Delta e} P^2$$

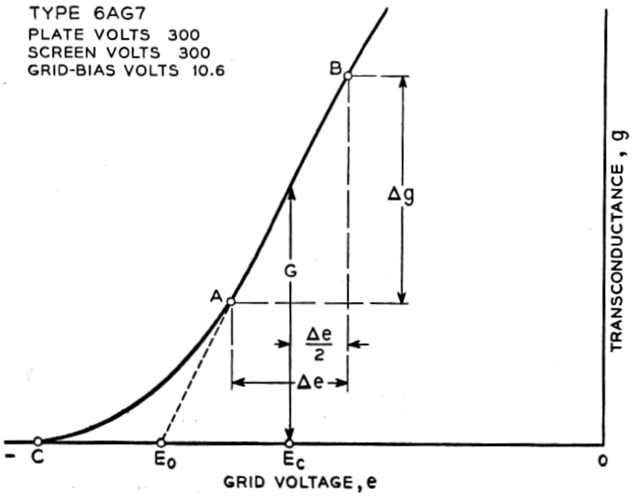

Fig. 1—*g-e* characteristic having principally second order modulation over part of the range.

To find the level H_2 of the second harmonic below the fundamental, prolong the straight portion of the characteristic down to the virtual cutoff E_0 (Fig. 1). Then

$$H_2 = 20 \log \frac{I_p}{I_{2p}} = 20 \log \frac{E_c - E_0}{P} + 12.$$

For the case when all of the parabolic characteristic is used

$$P = \frac{\Delta e}{2} \quad \text{and} \quad H_2 = 20 \log \frac{G}{\Delta g} + 18.$$

If it is desired to express H_2 in terms of the output current i_p and the space current i_0 the following approximate formula may be used:

$$H_2 = 20 \log \frac{i_0}{I_p} + 18.$$

This expression neglects the area under the characteristic on the left of the line AE_0, Fig. 1. The formula is useful in selecting a tube for closer consideration.

When a tube is used as a preamplifier in a wave analyzer an error of measurement may occur if two input frequencies intermodulate in the amplifier to produce a current of the same frequency as the one being measured. For instance, the fundamental $R \cos rt$ and the second harmonic $W \cos wt$ may intermodulate to form the third harmonic. If I_{r-w} is the disturbing current and I_p the wanted output, then

$$20 \log \frac{I_p}{I_{r-w}} = 20 \log \frac{E_c - E_0}{P} + 6 - 20 \log \frac{R}{P} - 20 \log \frac{W}{P}$$

The Rectifier

The portion of the plate current resulting from the rectification of a signal $P \cos pt$ is

$$I_{dc} = \frac{1}{4} \frac{\Delta g}{\Delta e} P^2.$$

If several frequencies were present, $P_1 \cos pt$, $P_2 \cos p_2 t$ and so on,

$$I_{dc} = \frac{1}{4} \frac{\Delta g}{\Delta e} (P_1^2 + P_2^2 + \cdots)$$

Thus I_{dc} is proportional to the square of the root-mean-square voltage input. This property of the parabolic tube of measuring the root-mean-square voltage is often useful in the measurement field.

If Δe is the parabolic range of the tube and Δg the corresponding change in g, the largest possible rectified current obeying the root-mean-square law will obtain for an amplitude $P = \frac{\Delta e}{2}$. Then

$$I_{\max} = \tfrac{1}{16} \Delta g \Delta e.$$

The Frequency Doubler

The second harmonic is given, as before, by

$$I_2 = \frac{1}{4} \frac{\Delta g}{\Delta e} P^2.$$

The largest possible output current is

$$I_{\max} = \tfrac{1}{16} \Delta g \Delta e.$$

In general, the level of the undesirable fundamental will be higher than the harmonic by

$$H_2 = 20 \log \frac{I_p}{I_{2p}} = \log \frac{E_c - E_0}{P} + 12.$$

For the maximum current case this reduces to

$$H_2 = 20 \log \frac{g}{\Delta g} + 18.$$

THE MODULATOR

When two inputs $P \cos pt$ and $Q \cos qt$ are applied to the grid, the output is

$$i = i_0 + \frac{1}{4}\frac{\Delta g}{\Delta e}(P^2 + Q^2) + \frac{1}{4}\frac{\Delta g}{\Delta e}(P^2 \cos 2pt + Q^2 \cos 2qt)$$
$$+ \frac{1}{2}\frac{\Delta g}{\Delta e} PQ \cos (p + q)t + \frac{1}{2}\frac{\Delta g}{\Delta e} PQ \cos (p - q)t$$

The last two terms represent the sidebands.

In the case of a detector the available supply of the carrier voltage Q is copious. Putting $Q = \frac{\Delta e}{2}$ the well known result is obtained

$$I_{p+q} = I_{p-q} = \tfrac{1}{4}\Delta g P.$$

The conversion transconductance is

$$G_c = \frac{\Delta g}{4}.$$

To formulate filtering requirements the signal and carrier leaks must be found.

The signal leak is

$$20 \log \frac{I_p}{I_{p\pm q}} = 20 \log \frac{G}{\Delta g} + 12.$$

The carrier leak is

$$20 \log \frac{I_q}{I_{p\pm q}} = 20 \log \frac{G}{\Delta e} + 20 \log \frac{\Delta e}{P} + 6.$$

In the design of a heterodyne oscillator a generous supply of both input voltages is easily available and maximum output current is desirable. This occurs when $P = Q = \frac{\Delta e}{4}$. Then

$$I_{p-q} = \tfrac{1}{32}\Delta g \Delta e.$$

Whether P equals Q or not, the unwanted products are

$$20 \log \frac{I_q}{I_{p-q}} = 20 \log \frac{E_c - E_0}{Q} + 6$$

$$20 \log \frac{I_p}{I_{p-q}} = 20 \log \frac{E_c - E_0}{P} + 6$$

$$20 \log \frac{I_{2p}}{I_{p-q}} = 20 \log \frac{P}{Q} - 6$$

$$20 \log \frac{I_{2p}}{I_{p-q}} = 20 \log \frac{Q}{P} - 6$$

$$20 \log \frac{I_{p+q}}{I_{p-q}} = 0.$$

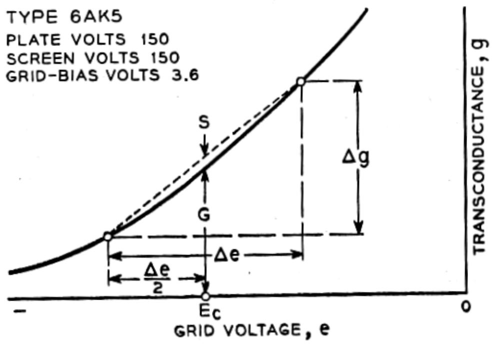

Fig. 2—*g-e* characteristic exhibiting third order modulation.

Third Order Modulation

When we do take into consideration the fourth term of equation (3), that is $\frac{1}{6} \frac{\partial^2 g}{\partial e^2} v^3$ the *g-e* characteristic will no longer be a straight line but a parabola. It turns out that all of the computations of the second-order effects as shown are so little affected that no corrections are necessary. The presence of third-order modulation caused by the term $\frac{1}{6} \frac{\partial^2 g}{\partial e^2} v^3$ will, however, add new types of modulation products to the output. These are usually objectionable.

A typical *g-e* characteristic with third-order modulation present is shown on Fig. 2. The curvature is usually concave upward. Instead of measuring the derivative $\frac{\partial^2 g}{\partial e^2}$ needed for the computations, it is more practical to scale off the amount S by which the characteristic sags in the middle of the interval Δe (Fig. 2).

The plate current is then given by the expression

$$i = i_0 + Gv + \frac{1}{2}\frac{\Delta g}{\Delta e}v^2 + \frac{4S}{3(\Delta e)^2}v^3. \tag{4}$$

SINGLE FREQUENCY INPUT

When the signal input to an amplifier consists of a single frequency $v = P \cos pt$, the output current is given by

$$i = i_0 + \frac{\Delta g}{\Delta e}P^2 + \left[G + \frac{S}{(\Delta e)^2}P^2\right]P\cos pt + \frac{1}{4}\frac{\Delta g}{\Delta e}P^2\cos 2pt + \frac{1}{3}\frac{S}{(\Delta e)^2}P^3\cos 3pt.$$

The second-order effects consisting of the rectified current and second harmonic are seen to be unaffected by the presence of third-order modulation. However, the first-order effect, the fundamental output, ceases to be linear and a new product, the third harmonic, appears.

The change in fundamental output is expressible as a loading effect on transconductance. The effective transconductance of the tube is.

$$G_e = G + S\left(\frac{P}{\Delta e}\right)^2.$$

Expressed in db's the non-linearity effect is approximately

$$20 \log \frac{G_e}{G} = 8.6\frac{S}{G}\left(\frac{P}{\Delta e}\right)^2.$$

When P is large it is convenient to select $\Delta e = 2P$ and get

$$G_e = G + \frac{S}{4}, \qquad 20 \log \frac{G_e}{G} = 2.15\frac{S}{G}.$$

S is positive when the g-e curve is concave upward.

The third harmonic content of the output is

$$H_3 = 20 \log \frac{I_p}{I_{3p}} = 20 \log \frac{G}{S} + 40 \log \frac{\Delta e}{P} + 10.$$

If we select $\Delta e = 2P$ the second term drops out

$$H_3 = 20 \log \frac{G}{S} + 22.$$

If the curve is concave upward the third harmonic increases the peak value of the wave. If it is concave downward the peak value of the wave decreases.

In the case of a two-frequency input $v = P \cos pt + Q \cos qt$ the second-order products are again unaffected. The third-order products will be:

$$I_{3p} = \frac{1}{3} \frac{S}{(\Delta e)^2} P^3$$

$$I_{3q} = \frac{1}{3} \frac{S}{(\Delta e)^2} Q^3$$

$$I_{2p \pm q} = \frac{S}{(\Delta e)^2} P^2 Q$$

$$I_{p \pm 2q} = \frac{S}{(\Delta e)^2} PQ^2.$$

There are situations in the design of detectors where the $I_{q \pm 2p}$ current may be disturbing. For example, in a wave analyzer modulation stage when measuring the second harmonic $P_2 \cos (2p)t$ the desired second-order product is $I_{q-(2p)}$. This is, however, of the same frequency as the third-order product I_{q-2p} generated by the intermodulations of the strong fundamental $P_1 \cos pt$ and the carrier $Q \cos qt$. The level of the wanted product with respect to the unwanted one is given by

$$20 \log \frac{I_{p-(2p)}}{I_{q-2p}} = 20 \log \frac{\Delta g}{S} + 20 \log \frac{\Delta e}{P_1} + 20 \frac{P_2}{P_1} - 6.$$

If there are two interfering inputs $R \cos rt$ and $W \cos wt$ they may, together with the carrier $Q \cos qt$, form an objectionable product $i_{r \pm w \pm q}$ of the same frequency as the wanted product i_{p-q}. The level of this disturbing product with respect to the wanted product is then given by

$$20 \log \frac{I_{p-q}}{I_{q \pm r \pm w}} = 20 \log \frac{\Delta g}{S} + 20 \log \frac{\Delta e}{R} + 20 \log \frac{P}{W} - 12.$$

When two input frequencies are present in the input, the effective transconductance of the tube becomes

$$G_e = G + \frac{S}{(\Delta e)^2} (P^2 + 2Q^2).$$

The amplifier gain depends on the level of all of the components of the input.

Fourth Order Modulation

If the fourth term of equation (3) is absent, but the fifth term $\frac{1}{24} \frac{\partial^3 g}{\partial e^3} v^4$ is present, fourth-order modulation will occur. The g-e characteristic will be a cubic with an inflection point at E_e.

TABLE I

If modulation up to the fourth order is present and $v = P \cos pt + Q \cos qt$, the plate current is $i = \sum_{m=0}^{m=4} \sum_{n=0}^{n=4} a_{mn} \cos (mp \pm nqt)$. The values of a_{mn} are:

m	n = 0	n = 1	n = 2	n = 3	n = 4
0	$\frac{1}{4}\frac{\Delta g}{\Delta e}(F^2 + Q^2) + \frac{3D}{4(\Delta e)^3}[4Q^2P^2 + Q^4 + P^4]$	$\left[G + \frac{S}{(\Delta e)^2}(2P^2 + Q^2)\right]Q$	$\left[\frac{1}{4}\frac{\Delta g}{\Delta e} + \frac{3D}{(\Delta e)^3}(P^2 + Q^2)\right]Q^2$	$\frac{1}{3}\frac{S}{(\Delta e)^2}Q^3$	$\frac{1}{4}\frac{D}{(\Delta e)^3}Q^4$
1	$\left[G + \frac{S}{(\Delta e)^2}(P^2 + 2Q^2)\right]P$	$\left[\frac{1}{2}\frac{\Delta g}{\Delta e} + \frac{3D}{(\Delta e)^3}(Q^2 + P^2)\right]PQ$	$\frac{S}{(\Delta e)^2}PQ^2$	$\frac{D}{(\Delta e)^3}PQ^3$	0
2	$\left[\frac{1}{4}\frac{\Delta g}{\Delta e} + \frac{3D}{(\Delta e)^3}(P^2 + Q^2)\right]P^2$	$\frac{S}{(\Delta e)^2}P^2Q$	$\frac{3}{2}\frac{D}{(\Delta e)^3}P^2Q^2$	0	0
3	$\frac{1}{3}\frac{S}{(\Delta e)^2}P^3$	$\frac{D}{(\Delta e)^3}P^3Q$	0	0	0
	$\frac{1}{4}\frac{D}{(\Delta e)^3}P^4$	0	0	0	0

$$g = G + \frac{\partial g}{\partial e} v + \frac{1}{6} \frac{\partial^3 g}{\partial e^3} v^3$$

To compute fourth-order effects a tangent is drawn to the curve in the middle of the range Δe (Fig. 3). It will represent a parabolic pentode over the range Δe and Δg. The departure D of the actual curve from the parabolic pentode at the extremes of the range Δe is measured. The g-e curve becomes

$$g = G + \frac{\Delta g}{\Delta e} v + \frac{8DV^3}{(\Delta e)^3}.$$

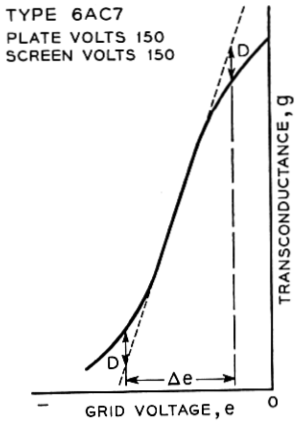

Fig. 3—g-e characteristic exhibiting fourth order modulation.

As a rule D is negative. It decreases the slope of the curve. The plate current is

$$i = i_0 + Gv + \frac{1}{2} \frac{\Delta g}{\Delta e} v^2 + \frac{2D}{(\Delta e)^3} v^4. \tag{5}$$

From this expression the fourth-order effects may be computed.

Single Frequency Input

Fourth-order modulation has no effect on the gain of the amplifier. It affects the second-order products, the rectified current and the second harmonic and adds the fourth harmonic.

In an amplifier it is unimportant to correct the value of H_2 for fourth-order modulation. The fourth harmonic is given by

$$H_4 = 20 \log \frac{G}{D} + 60 \log \frac{\Delta e}{P} + 12.$$

In a rectifier, the effect of fourth-order modulation is to destroy the square law of the rectifier. The error is

$$\frac{\Delta I_{dc}}{I_{dc}} = 3\frac{D}{\Delta g}\cdot\left(\frac{P}{\Delta e}\right)^2.$$

This correction is valid for the case of frequency doublers.

Two-Frequency Input

In a detector the fourth-order modulation produces an overloading effect; the modulator gain is then a function of the input

$$G = \frac{\Delta g}{4} + \tfrac{3}{8}D + \tfrac{3}{2}D\left(\frac{P}{\Delta e}\right)^2.$$

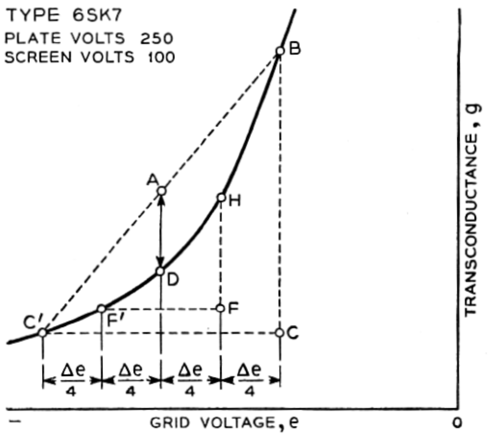

Fig. 4—g-e characteristic with second, third and fourth order modulation present. $D = \frac{2}{3}(\overline{CB} - 2\overline{FH})$; $\Delta g = \overline{CB} - 2D$; $S = \overline{AD}$

Neglecting $\frac{3}{8}D$ in comparison with $\frac{\Delta g}{4}$, the error in linearity is

$$\frac{\delta G_c}{G_c} = 6\frac{D}{\Delta g}\left(\frac{P}{\Delta e}\right)^2.$$

There will be also a cross-loading effect for an interfering signal $R \cos rt$.

$$\frac{\delta G_c}{G_c} = 2D\left(\frac{R}{\Delta e}\right)^2$$

In a heterodyne oscillator fourth-order modulation will produce a second harmonic at the output.

$$H_2 = 20\log\frac{\Delta g}{D} + 20\log\frac{\Delta e}{P} + 20\log\frac{\Delta e}{Q} - 10.$$

For the case of maximum output at $P = Q = \frac{\Delta e}{4}$

$$H_2 = 20 \log \frac{\Delta g}{D} + 14.$$

Accuracy of Computations

Before closing the subject, let us consider for a moment the accuracy involved. Very careful measurement of the characteristics of samples of some tubes, notably the 310A and 807 has shown that over a large range the fit with a parabola is excellent. On the other hand the electron tube bulletins give characteristics which are avowedly average. The drafting errors must be large and the temptation to use a straight edge instead of a french curve must be great. Probably no two observers will agree as to the extent of the straight part of a conductance curve. Even in the case of the 310A and 807 tubes mentioned above, manufacturing variations may affect the transconductance curve from tube to tube.

With this in mind we may conclude that the results obtained, that is, the values of the current obtained by the methods evolved above, must be approximate in character. The value of the analysis given lies in its simplicity rather than accuracy. Admitting that the situation is not satisfactory from the standpoint of reproducibility of results we must face the fact that there is no method nor the promise of a method for computing performance of a tube that would come within slide rule accuracy. We may console ourselves that in practice accuracy in estimating output levels and unwanted products is not really required. An error of 3 db or in the case of unwanted products an error of 6 db is of small consequence. This is about the variation to be expected to exist between two individual electron tubes and it must be included as a tolerance in determining performance requirements.

In connection with the third-order modulation the question arises whether the transconductance characteristic is really parabolic and not a curve of fourth or sixth degree and, if so, whether the equations derived above still apply.

While no accurate test can be applied conveniently we may compare a parabola with a quartic passing through the same three points. The parabola is characterized by a smooth curvature, while the quartic and the higher-degree curves have a flat middle portion with the ends turned up sharply. An inspection of the transconductance characteristics of tubes reveals that they are of the smoother curvature type—that is, that a square term is the chief contribution to the series representing the curve.

Should this not be the case and should we possibly mistake a quartic for a parabola, we still would obtain all of the phenomena caused by third-order modulation; that is, we would get a third harmonic, a transconductance increment, a finite detector discrimination, and so on, only their values would be somewhat different. Moreover, a more elaborate analysis reveals that the computations as made above would be in error by relatively small amounts and, what is more important, they would always be on the safe side. The only important error would be the absence of the fifth harmonic, which cannot be produced by third-order modulation.

Experience with computations checked by measurement reveals that the equations apply in a great majority of practical cases. The computations may be used, therefore, as a guide in the selection of tubes, operating parameters, and in experiments, even though we must realize that they may not be fully justified theoretically and are not quite accurate.

Conclusion

The analysis of the transconductance characteristics of tubes could be pushed further to fifth, sixth and higher orders of modulation, but it is hardly worth it. The mathematical treatment becomes burdensome, the results are uninteresting and the applications are rare except in a qualitative way. Thus, having completed the discussion of the fourth-order modulation, we find an extension of the treatment to higher orders of modulation unprofitable.

The material presented in this article has consisted chiefly of a list of formulas which may be applied to practical computations of the transmission and quality of transmission of electron tubes. They will be found to be useful in design of electron tube circuits. The second objective of the analysis is to give the user a mental picture of the tube performance in terms of its conductance characteristic.

Thus in general the quality of transmission of an amplifier, its discrimination against interfering voltages, amplitude distortion, and second harmonic content are measured by the bias cut-off interval. The capacity of the tube to deliver large currents at small distortion is measured by the area under the characteristic. The gain is measured by the transconductance and, in some cases, by the steepness of the characteristic. Freedom from third- and fourth-harmonic distortion and input-output linearity are measured by the linearity of the characteristic.

In a modulator the current capacity of the tube is measured by the area between the characteristic and the lines defining the voltage and the transconductance range used. The conversion transconductance is measured by the transconductance range available. The linearity of the characteristic will measure the discrimination against third-order products. In the case of a rectifier the criteria will be the same as in the case of the modulator.

A tube with a small slope and large transconductance will deliver with small distortion large currents whether used as an amplifier or modulator or rectifier. It will be a poor voltage amplifier, detector or voltmeter. A tube with a steep characteristic and large transconductance will be a good voltage amplifier, detector or voltmeter, but will carry little current.

Of course there are no rules without exception and special cases will be found in practice where the above recommendations may be violated. Electron tubes are used in many other ways than those discussed above and the present analysis covers only a small part of the field. The methods of computing modulation products shown above can be extended to triodes with little or no modification in cases of modulator or rectifier design where the external plate impedance is very low at the input frequencies.